Why we care about computation as astrophysicists [1]

"The purpose of computing is insight, not numbers." — Richard Hamming

The standard model of cosmology, i.e., the Λ–cold dark matter model, provides a remarkably successful description of the early Universe (Efstathiou et al., 1990; Ostriker & Steinhardt, 1995). Within this framework, the evolution of collisionless dark matter (DM) is governed by relatively simple physics. On non-relativistic scales, the equations of motion (EoM) of a DM particle only contains a gravitational term and a Hubble drag term. The bulk motion of these particles redistributes matter and alters the density field. When the fluctuations are small ($\delta \ll 1$), the equations can be linearized and solved analytically (Bernardeau et al., 2002).

These theories eventually fail as δ approaches unity, when particle trajectories become nonlinear. To move forward, Press & Schechter (1974) introduced an influential model of DM halo formation by assuming the idealized collapse of spherical overdensities. Later models replaced spherical collapse with ellipsoidal or triaxial collapse and obtained more accurate predictions for halo abundances (Sheth et al., 2001). Unlike linear perturbation theory, however, these models depend on additional assumptions that do not converge toward the reality in a well-defined limit.

The need for such assumptions reflects the difficulty of solving the nonlinear system analytically. In a gravitational N-body system, the EoM of each particle is coupled to those of all other particles. Except in special configurations, the variables cannot be separated, and no general closed-form solution exists even for $N = 3$ (e.g., Stone & Leigh, 2019). Thus, *even when the microscopic laws are known precisely, deriving the macroscopic behavior of the system remains analytically intractable*[2]. This limitation is catastrophic because it prevents theorists from analytically deriving complex behaviors that are not apparent from the EoM of any individual particle. The formation of DM halos, filaments, and voids emerges from the collective nonlinear motion of a large number of particles—*more is different* (Anderson, 1972).

Numerical methods offer a new way to resolve this limitation. Running on general-purpose computers, these methods replace a continuous problem with a finite-element representation, approximating differential equations by discretized update rules. For a Riemann integrable system, the numerical solution should approach the true solution as the resolution improves (Lax & Richtmyer, 1956). Numerical experiments began to revolutionize the

[1]This article is taken from Yingtian Chen's PhD thesis (§1.1).

[2]But this is not impossible. In a recent breakthrough, Deng et al. (2025a,b) rigorously derived the hydrodynamic equations (macroscopic) and Boltzmann's kinetic equation (mesoscopic) from Newtonian dynamics (microscopic). Yu Deng received a Fields Medal in 2026 while this thesis was being written.

study of structure formation in the 1970s. Although the earliest experiments contained only hundreds of particles, they already revealed nonlinear gravitational clustering and provided tests of competing analytical models (Verlet, 1967; Press & Schechter, 1974; Evrard, 1988; Bertschinger, 1998). We now call these "experiments" as "simulations", where each computer program constructs a mini universe governed by its own physical laws.

Similar breakthroughs occurred across many fields, from the chaotic behavior of the Navier–Stokes equations (Lorenz, 1963), phase transitions in lattice models (Metropolis et al., 1953) to the emergence of intelligence from neural systems (Hopfield, 1982); see Fig. 1 for the current state-of-the-art of numerical models across sciences. *Expecting simple laws in complex systems is the arrogance of theorists; Expecting no law at all is the shortsightedness of experimentalists*[3]. Computation, on the other hand, connects simple laws to complex worlds, allowing researchers to move beyond the limits of human intuition.

[3] Quoted from Ziming Liu, the author's college classmate.

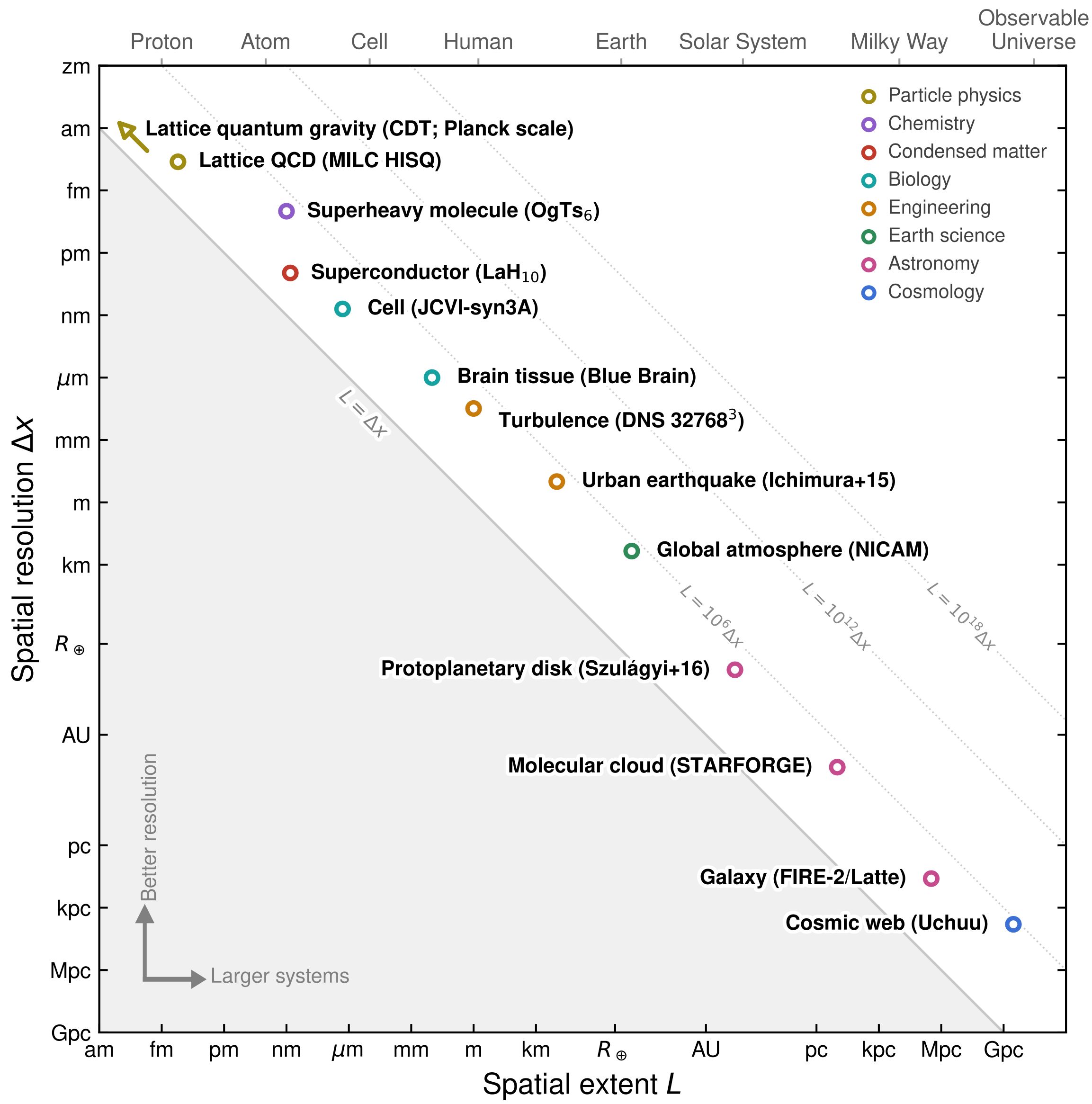

Figure 1: Spatial extent L vs. resolution Δx for 13 selected state-of-the-art numerical models across sciences. L is either the box size for volume-filled simulations or the size of the largest resolved object for isolated objects. Δx is either the cell spacing or median particle separation for particle-based simulations. Reference objects are listed along the top. The gray region below $L = \Delta x$ is unphysical, and dotted lines mark constant dynamic range. Objects simulated are quantum spacetime (Loll, 2020), QCD vacuum (Bazavov et al., 2013), a superheavy molecule (Malli et al., 2021), a hydride superconductor (Errea et al., 2020), a minimal cell (Stevens et al., 2023), brain tissue (Markram et al., 2015), homogeneous turbulence (Yeung et al., 2025), a city under seismic loading (Ichimura et al., 2015), Earth's atmosphere (Satoh et al., 2014, 2025), a protoplanetary disc (Szulágyi et al., 2016), a molecular cloud (Grudić et al., 2021), a Milky Way-mass galaxy (Wetzel et al., 2016; Hopkins et al., 2018), and the cosmic web (Ishiyama et al., 2021).

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